1901MNRAS..62..132H

irregular, there will be opportunities for selecting pairs of stations accurately in the line of march, and 100 feet apart (or some other convenient distance). By pairs of photographs taken as above the positions of near objects can be determined well, and those of distant objects not so well. But these latter will appear on many photographs, and we thus get longer bases from which to determine them more accurately. A compass needle attached to the camera, though useless for avoiding minute differences of orientation, would be very useful in comparing results at distant stations.

On the Accuracy of Measures on Photographs: Remarks on recent Papers by M. Loewy and Mr. H. C. Plummer. By Arthur R. Hinks, M.A.

In the supplementary number of the *Monthly Notices*, 1901 (vol. lxi., p. 618), Mr. H. C. Plummer has recently discussed the very elaborate memoir by M. Loewy, "Sur la précision des mesures des coordonnées rectilignes des images stellaires," in the eighth circular of the *Conférence astrophotographique internationale de Juillet* 1900. M. Loewy has sought to find a general formula which shall give the probable error of the mean of a number of settings upon a star image, in a number of different orientations of the plate; and concludes that to avoid systematic error it is necessary to measure the plate in four orientations. Mr. Plummer has rediscussed the figures which M. Loewy gives, and comes to a number of conclusions, of which the most important are:

"(4) By combining measures made in the direct and reversed positions of the plate a much better result is obtained, to which M. Loewy's formula does *not* apply.

"(6) The error eliminated by reversing the plate is definite for each image, but may vary both in magnitude and in sign from one star to another."

Briefly, the principal point in dispute is, Can a sensible compensation of the systematic errors of measurement be obtained by measuring the plate in two positions whose orientations differ by 180°?

My excuse for intruding into the discussion between M. Loewy and Mr. Plummer at this point is as follows: For some months I have had the pleasure of being in correspondence with M. Loewy in reference to certain aspects of the question; he has been so kind as to give me very full and interesting explanations of his views; and I have his permission to give some passages from his letters in the present note.

At the beginning of his memoir (*loc. cit.*) M. Loewy says:

"Il y a donc à distinguer, lorsqu'on détermine une co-

1901MNRAS..62..132H

ordonnée rectiligne, deux catégories d'erreurs : l'une, l'erreur accidentelle E_a, et l'autre, E_s, l'erreur systématique. . . . L'erreur totale E_t se trouve donc exprimée par la formule

$$E_t = \sqrt{E_a^2 + E_s^2}. \text{ ''}$$

Now it is evident at once, as Mr. Plummer has pointed out, that the expression "systematic" error is not used here in its ordinary sense ; truly systematic errors of course do not follow the law of distribution which is the basis of the method of least squares. And to destroy the validity of the above equation it would be necessary to show only that the distribution of error in photographic measures is not entirely accidental ; that it tends to be of opposite sign in opposite orientations of the plate for example.

On this point M. Loewy writes (1901 October 16) :

"Je saisis cette occasion de préciser d'avantage la principe qui m'a guidé dans l'établissement des formules.

"Une fois la moyenne d'une série de pointés formée, ce résultat est affecté de deux catégories d'incertitudes, l'une a pour expression E_a/n" [apparently a slip for $E_a/\sqrt{n}$] "et l'autre est représentée par E_s.

"E_s est une erreur accidentelle d'une autre espèce affectant la moyenne. . . .

"Or, les expériences ont montré que, lorsqu'on répète les mesures dans une même orientation, avec un même observateur, après plusieurs mois d'intervalle, E_s varie notablement. Pour ce motif, on préfère ne répéter une seconde fois la détermination des coordonnées rectilignes que plusieurs mois après l'exécution de la première série de mesures. . . . Il n'est permis ici de faire aucune supposition sur une loi quelconque qui relie entre elles les différents valeurs de E_s, soit en signe, soit en grandeur. E_s est évidemment à considérer comme une erreur accidentelle d'un autre genre.

"Comme il convient, d'un côté, d'attribuer le maximum d'effet aux sources d'erreurs, et qu'il fallait d'un autre côté embrasser la généralité des cas, j'ai dû admettre qu'il n'existe aucun lien entre les E_s relatives, aussi bien à différents observateurs qu'à diverses orientations, même si celles-ci diffèrent de 180°—ayant fait d'ailleurs, comme je l'ai dit dans ma dernière lettre, des expériences qui confirment cette dernière hypothèse."

These experiments are thus described in a former letter (1901 September 12) :

"On a généralement admis, ainsi qu'à Paris, que cette petite anomalie constante dans une orientation donnée, s'élimine dans la moyenne de deux séries de mesures faites dans des orientations différant de 180°. Comme cela a été expressément répété dans plusieurs endroits de ma note, on ne peut admettre *a priori* aucune hypothèse, quelque soit son degré de probabilité, lorsqu'il s'agit d'atteindre le dernier degré de précision.

1901MNRAS..62..132H

"Nous avons fait sur cette question de nombreuses expériences, avec divers appareils, et dans des conditions variées, et j'ai reconnu que dans le plus grand nombre des cas cette supposition est absolument inadmissible. On arrive *à peine* à une compensation partielle d'erreurs que dans des cas exceptionnels. . . .

"Dans le cliché que nous avons examiné, grace à ces qualités, il s'est produit dans les nombres relatifs aux positions différant de 180° quelques compensations; mais elles sont de peu d'importance, en regard de la somme totale des résidus. En effet en formant d'abord deux à deux la moyenne des résidus entre les positions différant de 180°, et calculant après l'erreur probable, on trouve une valeur à peine différente d'un centième de seconde d'arc de celle figurant dans ma note."

To sum up briefly M. Loewy's view, it is necessary to consider "l'erreur systématique" as an accidental error of another kind, that is to say, differing from the accidental error deduced from the divergence of successive pointings in a single orientation, because, in the first place, experiments have shown that the systematic error is certainly not really systematic, but varies from time to time, and secondly because a rediscussion of the observations on the supposition that there may be some compensation when the plate is reversed leads to the original result. Now this is in absolute contradiction to the result which Mr. Plummer deduced from the same figures. And the discrepancy is of the highest importance because the validity of M. Loewy's fundamental equation turns upon it. If the "accidental error of another kind" can be shown to be not really accidental, if the errors in the measures of a given image in a given orientation of the plate are not equally likely to be positive or negative, then E_s is not an error which comes within the operation of the ordinary theory of errors, and the whole of M. Loewy's theory which follows on his fundamental assumption breaks down.

In a case such as this, where the validity of the application of the method of least squares is in dispute, there is at any rate a formal advantage in examining it from a point of view which does not involve the deduction for after-comparison of probable errors by the usual laws. I have therefore examined a part of M. Loewy's figures as follows.

His plate has a triple exposure, and 82 triple images were measured by two observers, A and B, in four orientations. Both M. Loewy and Mr. Plummer have treated the means for the three exposures as one complete measure. There is, however, a certain advantage in considering the exposures separately. Again, both have discussed the residuals of each setting from the mean of all for the two observers. Apart from the question which Mr. Plummer raises whether the two observers are equally skilled, there is a curious discontinuity in the results for B that leads to the suspicion that parts of certain columns of his figures have been interchanged. I fully agree with Mr. Plummer that this discontinuity is manifest—it appears in the measures for the

1901MNRAS..62..132H

separate exposures, as well as in the mean for the three; and I propose therefore to discuss only the measures of observer A.

In Tables VI. 1 and VI. 2 of M. Loewy's paper we find the individual measures made upon the first and second images of each of the 82 stars, in four orientations, 0°, 90°, 180°, and 270°. Forming afresh the means for observer A only, we obtain from his measures 656 residuals. The point to be determined is whether these tend to be of opposite signs in opposite orientations. We will consider separately the pairs of opposite orientations 0° and 180°, 90° and 270°, and divide each series into two classes—I., in which the residuals in opposite orientations are of the same sign; II., in which they are of different signs. If the residuals are distributed impartially, both as to sign and magnitude, these two classes will be equal. We will further subdivide each class according to the magnitude of the sums of the pairs of residuals, without regard to sign. Again, if the residuals are distributed impartially, large numerical sums will occur with equal frequency in each group.

The results of this analysis are as follows:

Classification of Numerical Sums of Residuals in Opposite Orientations.

	0° and 180°.		90° and 270°.	
Exposure 1.	I. Same Signs.	II. Opposite Signs.	I. Same Signs.	II. Opposite Signs.
Sums between 0 and 19	13	7	7	9
20 ,, 39	7	16	8	24
40 ,, 59	8	12	9	13
60 ,, 79	2	7	2	5
80 ,, 99	1	3	1	2
100 and over	0	6	0	2
	31	51	27	55
Exposure 2.				
Sums between 0 and 19	13½	9½	14½	9½
20 ,, 39	13½	18½	15	13
40 ,, 59	3	6	3	14
60 ,, 79	0	7	0	9
80 ,, 99	0	4	0	2
100 and over	1	6	1	1
	31	51	33½	48½

The halves appear because in some cases one residual is zero, and the pair has to be divided between groups I. and II.

These numbers give abundant support to two conclusions:

(1) That in pairs of opposite orientations the residuals are more frequently of opposite sign than of the same sign.

(2) That when the residuals are large, and their numerical sum correspondingly large, they are almost always of opposite sign.

1901MNRAS..62..132H

It seems to me, then, that these residuals cannot be treated as accidental in any sense whatever; that they do not come under the operation of the ordinary law of accidental errors at all; and that consequently M. Loewy's fundamental equation is not valid. I do not therefore find myself in agreement with Mr. Plummer when he says: "We may follow M. Loewy in denoting the accidental and systematic errors by E_a and E_s respectively, and the total probable error by E_t, and admit that $E_t^2 = E_a^2 + E_s^2$." This is indeed in obvious contradiction to his conclusion that there is a striking compensation of errors when the plate is measured in reversed positions—a conclusion which is entirely supported by the figures given above.

One can scarcely avoid coming to the conclusion that it will be wise to hesitate before committing oneself to the arduous programme which M. Loewy recommends; and this view is supported by the circumstance that there is another category of error, of which he has not up to the present time taken any account, the real error of position of the star image on the plate, due to such causes as irregularity in the size or distribution of the particles of silver in the image, or local shrinkage of the gelatine film which is not eliminated by the use of the réseau. It is not possible to discuss the magnitude of this source of error on M. Loewy's plates, because he has given the measures of one coordinate only, and one cannot discover whether there has perhaps been a slight change of orientation, &c., in the interval between the exposures which would affect the differences between the coordinates with an error which is a linear function of those coordinates. It has, however, been abundantly proved that this third category of error may exist; it is not possible to deduce an estimate of the real accuracy of the resulting positions of the star from an examination of the discrepancies between the measures on a single exposure, or set of exposures treated as one, however carefully the conditions of measurement may be varied. The ultimate test is the agreement between the results of two separate exposures when one is rigidly reduced to be comparable with the other, if necessary. And it should be noted that a really systematic error in measurement, if such exists, unchanged from star to star, is of no moment, because it is eliminated by the process of finding from known stars the correction to the plate centering.

It seems to me, then, that the following process gives an estimate of the probable error of a complete measure which is a trustworthy guide to the real accuracy with which relative star places can be obtained from a photograph. Take two exposures immediately following one another: measure them, and examine (1) whether the differences between the measured coordinates of the two exposures depend upon the position of the star upon the plate; (2) whether they vary with the magnitude of the star, as they may possibly do if the guiding is not perfect. If they pass this examination, the divergences of the differences from the

1901MNRAS..62..132H

mean will be a criterion of the inaccuracy which arises from all those categories of error, accidental error of setting, systematic error of measurement due to personality or otherwise, and systematic error of position of image due to irregularity or local defects. It is, then, of no consequence to examine elaborately the divergences from the mean of settings on a single exposure.

Cambridge Observatory:
1901 *December* 12.

The Spectrum of Nova Persei from 1901 *February* 28 *to April* 26. *With Appendix on the Spectrum in September.*

Rev. W. Sidgreaves, S.J., Stonyhurst College Observatory.

Introduction.

In "Notes on the Spectrum of *Nova Persei*," presented at the March meeting of the Society, it was briefly stated that the notes referred chiefly to photographs obtained with an objective prism of 4 inches aperture mounted on the finder of the large equatorial. It is necessary to preface the present paper by a more detailed description of this prismatic camera, on account of the difficulties which had to be overcome in order to secure accurate settings of the spectrum on the photographic plate. The importance of this adjustment is many times greater when the object-glass is corrected only for the visual rays than when a correcting lens for the photographic rays is employed, on account of the steep inclination of the plate to the optic axis of the lens, required for the simultaneous focus of the several photographic rays.

The camera is provided with a rotating frame for carrying the dark slide, and a graduated circle for setting the angle of inclination required. Another slide of the same dimensions as the dark slide carries three platinoid wires drawn taut over screw-heads—a pair of parallel wires, and one at right angles to the pair. When this slide is in the camera the wires lie in the plane to be occupied by the photographic plate. Its structure is simple, consisting of a plate of hard wood, with a central hole over which the wires are drawn, and side rails bringing up the sliding thickness to that of the dark slide. The space between the rails is left open at the ends to admit the light of a taper to illuminate the wires.

When the star is in the field its spectrum is seen with an eyepiece at the back of the camera as a sharp colour line, red at one end and blue at the other, together with the wires. The camera is set in position with reference to the prism by turning it in its adapter to bring the pair of lines parallel to the colour line image of the star; and the spectrum is brought into position on the